The Astrolabe

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Foreword

Aim of this treatise on the astrolabe is to discuss the mathematical principles laying beneath this astronomical instrument. This in order to reconstruct the astrolabe of the Persian astronomer and mathematician Al-Khujandī for an observer at general latitude. Therefore, this paper could be quite hard for someone who does not have a mathematical background. Nevertheless, it tries to provide a clear explanation of the anatomy and use of the astrolabe. The mathematical way of writing enforces respect to the knowledge of scientists from more than one thousand years ago. A replica of the astrolabe of Al-Khujandī is displayed in the Istanbul Museum for the History of Science and Technology in Islam.

The Istanbul Museum for the History of Science and Technology in Islam has been established in 2008. Its collection consists of replicas of the artifacts which are displayed at the *Institut für Geschichte der Arabisch-Islamischen Wissenschaften* in Frankfurt of prof. dr. F. Sezgin.

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1 The History and Purpose of the Astrolabe

One of the most widely used astronomical instruments over time is the astrolabe. Astrolabes were mainly used for two purposes: for astronomical observations and calculations, and for astrology. In this treatise we focus on the astronomical purposes of the astrolabe.

The astrolabe is a flat and portable instrument mostly made of brass on which the positions of the sun and some of the major stars are projected relatively to the observers horizon. Its mathematical principles date back to Greek antiquity. The principle of stereographic projection was known to the Greek astronomer Hipparchus of Nicaea (150 B.C.). Although it remains uncertain, it seems probable that Ptolemy used an instrument like the astrolabe. Certain is that he wrote a treatise on stereographic projection. Unfortunately no Greek astrolabes have survived. A new impulse to the astrolabe was given by Islamic astronomers. Not only did they translate Greek works on the astrolabe to Arabic, also they added new features. The azimuthal lines on the plates of the astrolabe were attached, to measure the direction of the sun or the stars. The 8th century Persian astronomer Muhammad Al-Fazari is credited to have built the first astrolabe.

Islamic astronomers desired a very precise measurement of time for their observations in days that reliable clocks were not yet available. An astrolabe suffices in that desire. It can be used to measure time in a very precise way, at day as well as at night, using the trajectory of the sun or the stars along the sky. Also the direction of the sun or the stars could be measured, using the azimuthal lines drawn on the plates of the astrolabe. Astrolabes were used to find the times of sunrise and the rising of fixed stars, to help schedule prayer times. It could also be used for determining heights of buildings or mountains, or depths of wells for instance. When knowing the longitude one was located, one could also determine *qibla*, the direction of prayer to Mecca, although an astrolabe was not used for this purpose. As an astrolabe was not used to navigate, albeit many papers state it as one of the most important features of an astrolabe.

Abu Al-Zarqali of Al-Andalus constructed an astrolabe which, unlike its predecessors, did not depend on the latitude of the observer, and could be used anywhere. Islamic astronomers also constructed different forms of the astrolabe. The linear astrolabe was an astrolabe in the shape of a stick. It was very difficult to use and to understand, which was also the reason it was rarely made. Also the spherical astrolabe does not seem to be very succesful. Although it was instructive to project the celestial bodies on a sphere, the flat astrolabe, or mostly called, the planispheric astrolabe was far more often used. The planispheric astrolabe is what we simply call the astrolabe.

Approximately one thousand astrolabes have survived time. Most of them are Islamic astrolabes and instruments from Christian Europe. In Europe astrolabes were used from the 11th century onwards, as the astrolabe was introduced via Islamic Al-Andalus to Latin Europe. It was in Al-Andalus that treatises on the astrolabe were translated from Arabic to Latin, and henceforth the knowledge spread slowly over Europe. It is worth noticing that the star names were just Latinized, so the stars preserved their original Arabic names. The popularity of the astrolabe was in Europe at its height during the 15th and 16th century. Astrolabes were used there until the 17th century, when other accurate devices to measure time were invented, like the pendulum clock by Christiaan

Huygens. In the Islamic world however the astrolabe remained popular until the 19th century.

For more on the astrolabe please consult the catalog of F. Sezgin [2], the book by D.A. King [3] or the book by J. Evans [1].

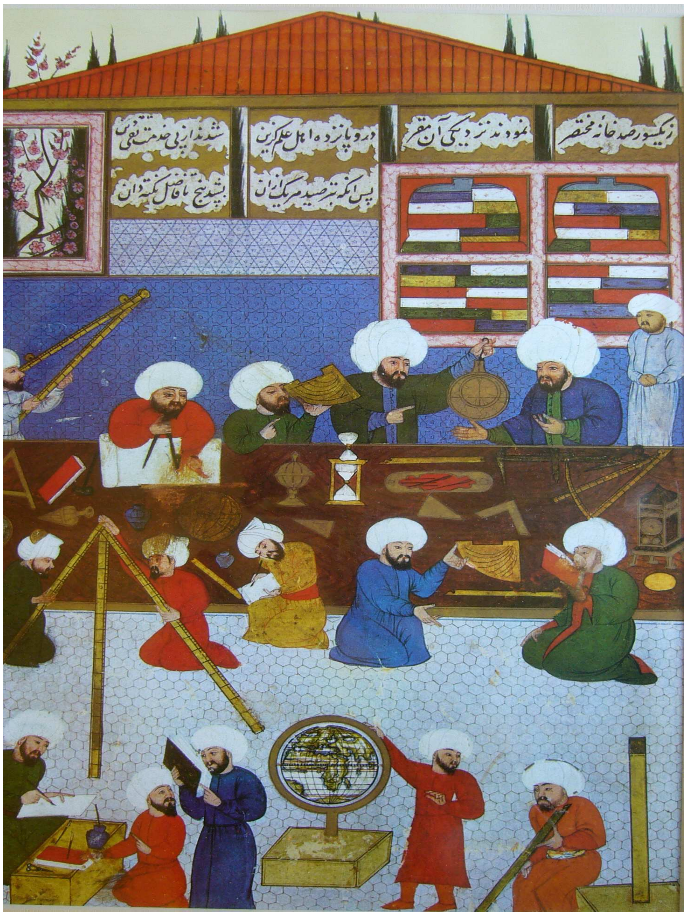

Figure 1: Taqī al-Dīn using an astrolabe in his Istanbul observatory.

2 The Anatomy of the Astrolabe

In this section we derive formulas for the construction of an astrolabe similar to the brass astrolabe of Al-Khujandī. The Persian astronomer and mathematician Abū Maḥmūd Ḥāmid ibn al-Khiḍr Al-Khujandī was born in nowadays Tajikistan, in the city of Khujand, and worked at the observatory of Baghdad. He constructed his astrolabe in the year 984/985 A.D. It is one of the oldest astrolabes preserved. It is currently exhibited in a museum in Doha, Qatar.

An astrolabe is a set of closely fitting brass parts. The main part of the astrolabe is

Figure 2: Frontside of the fully assembled astrolabe of Al-Khujandī, with the characteristic lions on its throne.

called the *mother* - *umm* in Arabic or *mater* in Latin. This thick brass plate has a centre hollowed out to leave a raised outer rim on its edge. A ring is attached to the *throne* of the astrolabe. The astrolabe of Al-Khujandi is famous for the lions on its throne. On the outer rim a scale from 0 to 360 degrees is engraved. The zero point of the scale is set on top of the astrolabe, under the throne. On astrolabes from Islamic astronomers Arabic abjad numbering is used. In this numberal system the 28 letters of the Arabic alphabet are assigned numerical values, see Table 1. On the astrolabe of Al-Khujandī the degrees are written for every five degrees on the outer rim. For convenience only the number five is written on the rim, when a number is ending with a five.

1	ا	12	يب	120	قك
2	ب	20	ك	200	ر
3	ج	30	ل	300	ش
4	د	40	م	400	ت
5	ه	50	ن	500	ث
6	و	60	س	600	خ
7	ز	70	ع	700	ذ
8	ح	80	ف	800	ض
9	ط	90	ص	900	ظ
10	ي	100	ق	1000	غ

Table 1: The abjad numbers. The number 12 is a combination of 10 and 2.

2.1 The Plates

The astrolabe is based on the mathematical principles of the *celestial sphere* and *stereographic projection*. The celestial sphere is an imaginary sphere concentric with the Earth on which the stars and the apparent path of the sun are projected from the center of the Earth. Stereographic projection is a method to map a sphere onto a plane, in this case the celestial sphere is mapped from the celestial south pole onto the plane of the celestial equator. Stereographic projection has the nice property it projects lines and circles onto lines and circles, and not onto ellipses or hyperboles, for instance. This is the reason why astrolabes could relatively easy be constructed, in principle it could be done by just using a ruler and a compass.

The hollowed out mater leaves enough space for several *plates*. These plates display (parts of the) the stereographic projections of the following points and circles:

- The centre of the plate is the *celestial north pole*, which is the centre of three concentric circles: the *Tropic of Cancer*, the *celestial equator* and the *Tropic of Capricorn*.
- The *horizon*, which projection is visible on the plate in Eastern, Northern and Western directions. The *twilight line* is 18 degrees below the horizon.
- The *almuqanṭarāt* (altitude circles) are the nearly concentric circles 3, 6, 9, ... degrees above the horizon.
- The *zenith* is the point directly above the head of the observer, i.e. 90 degrees above the horizon. The point directly beneath the observer, and thus opposite to the zenith, is called *nadir*.
- The *azimuthal circles* or circles of equal direction. Its projections are drawn for 5 degree intervals and are numbered at their intersections with the horizon. The *first vertical* is the azimuthal circle through the East and the West point. It is the reference circle for the other azimuthal circles. Note that all azimuthal circles pass trough the zenith.

Since the projections change according to the latitude of the observer, there were sometimes up to six plates in one astrolabe, to leave the user to pick the most appropriate for his latitude.

2.2 Reconstruction of the plate

For the reconstruction of the plate, let us first focus on how the equator, the Tropic of Capricorn and the Tropic of Cancer of the celestial sphere are projected onto the plane of the celestial equator. These projections are not dependent on the latitude of the observer.

It is obvious that the celestial equator is projected onto itself. So the projection of the celestial equator is centered around (0,0) with radius R, the radius of the celestial

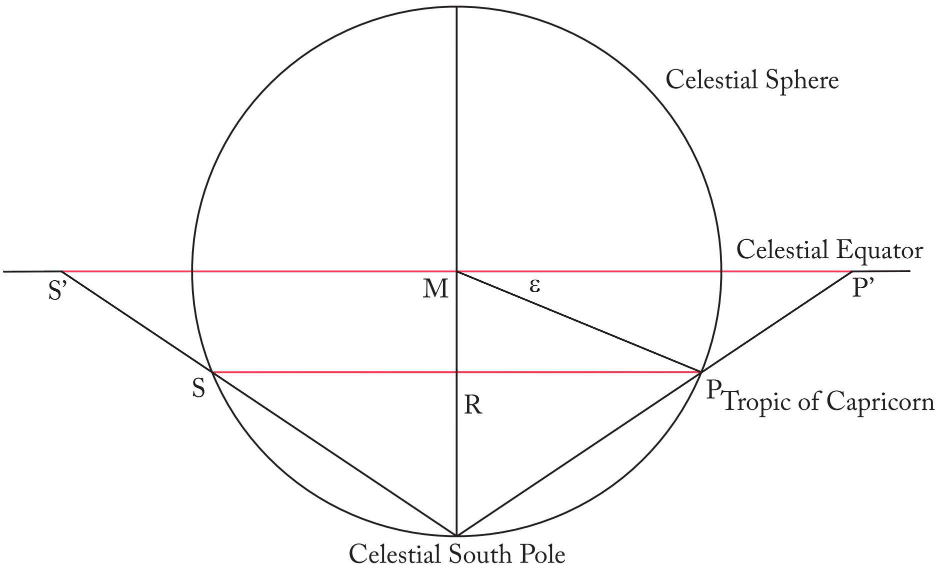

Figure 3: Stereographic projection of the Tropic of Capricorn onto the plane of the celestial equator, in cross section. The points P and S on the Tropic of Capricorn are respectively projected to the points P' and S' in the plane of the celestial equator. The distance between M and P' is $R\tan(45° + \epsilon/2)$.

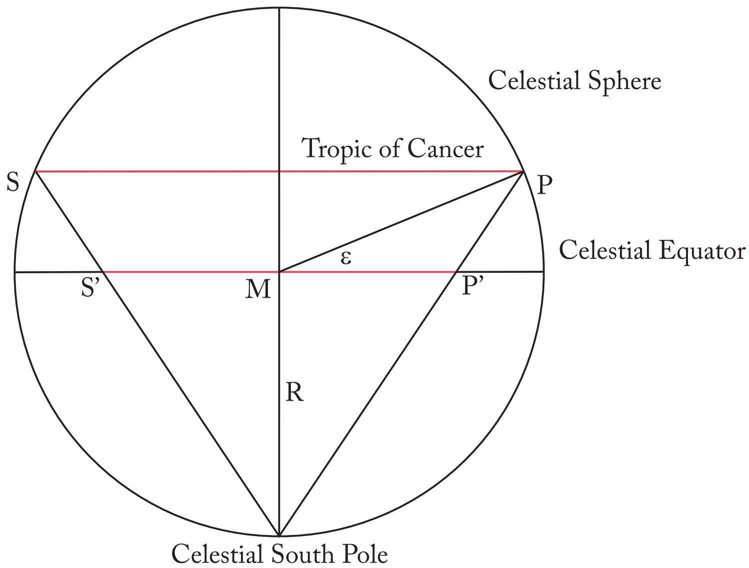

Figure 4: Stereographic projection of the Tropic of Cancer onto the plane of the celestial equator. The distance between M and P' is $R\tan(45° - \epsilon/2)$.

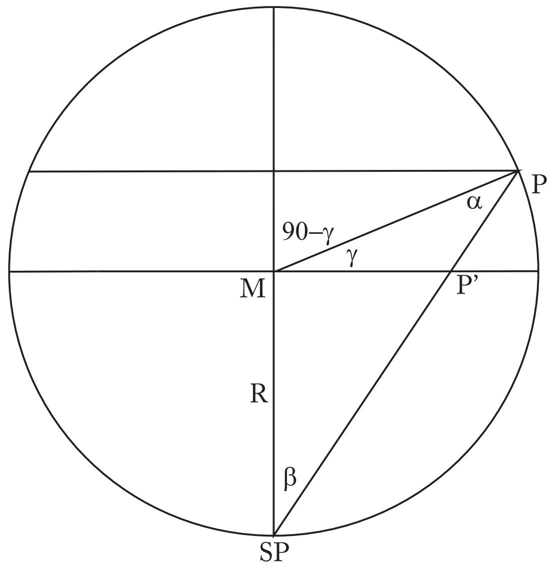

Figure 5: Angles for circles in the Northern Hemisphere.

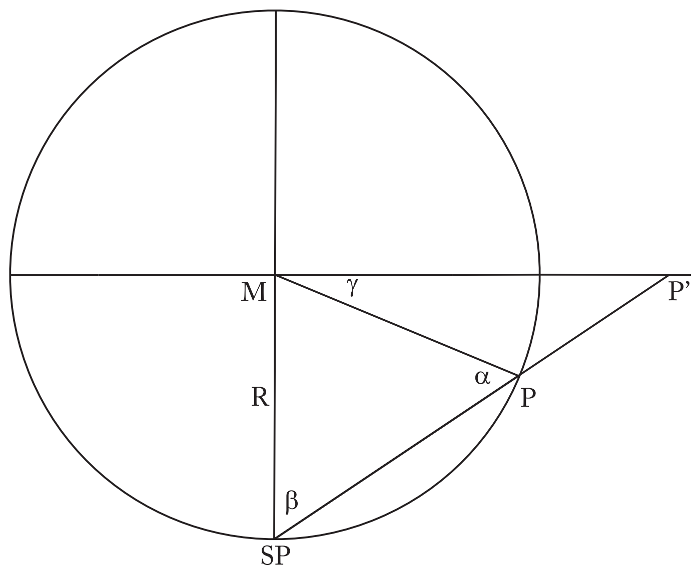

Figure 6: Angles for circles in the Southern Hemisphere.

equator. The rectangular coordinate system we use for the plate is orientated such that the meridian is projected on the y-axis. The South point of the projection of the horizon has positive y-coordinate. The x-axis is oriented such that the projection of the West point of the horizon has positive x-coordinate.

The Tropic of Capricorn is projected onto a circle in the plane of the celestial equator with center (0,0) and radius $R\tan(45° + \epsilon/2)$, see Figure 3. The projection of the Tropic of Cancer is also centered around (0,0), with radius $R\tan(45° - \epsilon/2)$, see Figure 4.

Here ϵ is the obliquity of the ecliptic, the apparent one year path of the sun around the Earth. Therefore the Tropic of Cancer is the most northerly latitude at which the sun can appear directly overhead at noon. Likewise, the Tropic of Capricorn is the most southerly latitude.

For the calculation of the radius of the projected circles, we take a closer look at Figure 5 and Figure 6. From there we see that it holds for the angles, in case a circle in the Northern Hemisphere is projectied, that

$$90° + \gamma + \alpha + \beta = 180°.$$

Here we have used that the angles in a triangle sum up to 180 degrees. In case the circle is in the Southern Hemisphere we have the relation:

$$(90° - \gamma) + \alpha + \beta = 180°.$$

Since it holds for the angles α and β that they are equal, i.e. $\alpha = \beta$, we have that it holds for circles in the Northern Hemisphere that $\beta = \frac{1}{2}(90° - \gamma)$, resulting eventually in a distance between M and P' in Figure 5 of

$$MP' = R\tan\frac{1}{2}(90° - \gamma).$$

For circles in the Southern Hemisphere we have that $\beta = \frac{1}{2}(90° + \gamma)$, resulting in a distance between M and P' in Figure 6 of

$$MP' = R\tan\frac{1}{2}(90° + \gamma).$$

2.3 Projection of an altitude circle

Let us now focus on the projection of the horizon as seen by an observer at latitude ϕ. From Figure 7 it is seen that it holds that

$$MA = R\tan\frac{1}{2}\phi$$

and that

$$MB = R\tan\frac{1}{2}(180° - \phi) = R\tan(90° - \frac{1}{2}\phi).$$

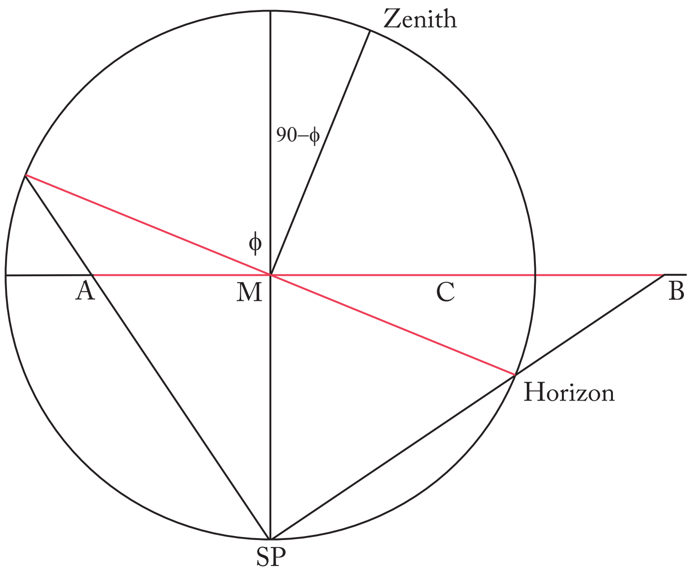

Figure 7: Projection of the horizon for an observer at latitude ϕ.

Henceforth, the center C of the projection of the horizon is located at

$$C = \left(0, \frac{R}{2}\left[\tan(90^\circ - \frac{1}{2}\phi) - \tan(\frac{1}{2}\phi)\right]\right).$$

Now we use the trigonometric identities

$$\tan(90^\circ - \theta) = \frac{1}{\tan\theta}, \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \tag{2.1}$$

to write that

$$\tan(90^\circ - \frac{1}{2}\phi) = \frac{1}{\tan\frac{1}{2}\phi},$$

and, while substituting $\phi/2 = z$, for the y-component of C, that

$$\frac{1}{2}\left(\frac{1}{\tan z} - \tan z\right) = \frac{1-\tan^2 z}{2\tan z} = \frac{1}{\tan 2z}.$$

Then we have for the center C that

$$C = \left(0, \frac{R}{\tan 2z}\right) = \left(0, \frac{R}{\tan\phi}\right).$$

Let us now consider an altitude circle a degrees above the horizon. Let us first focus on the case that $a < \phi$. Then we see from Figure 8 that it holds that

$$MA = R\tan\frac{1}{2}(\phi - a)$$

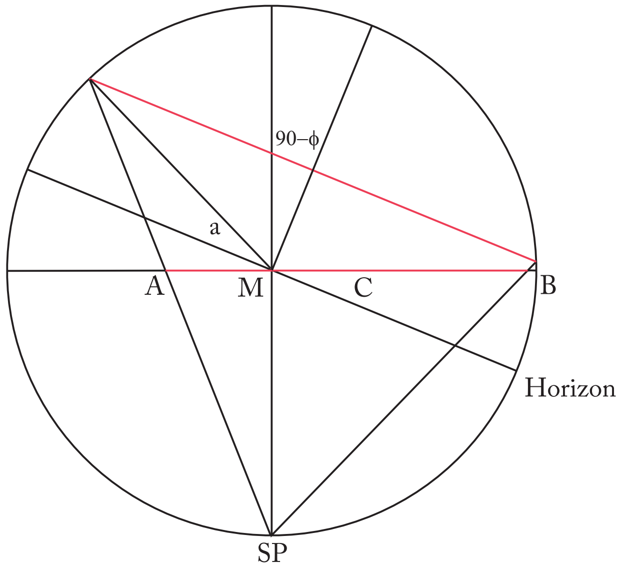

Figure 8: Projection of the altitude circle $a < \phi$ degrees above the horizon for an observer at latitude ϕ.

and that

$$MB = R \tan \frac{1}{2}(180^\circ - \phi - a).$$

Henceforth the center C of the projection is located at

$$C = \left(0, \frac{R}{2}\left[\tan(90^\circ - \frac{1}{2}(\phi + a)) - \tan \frac{1}{2}(\phi - a)\right]\right). \tag{2.2}$$

In case the altitude circle is $a = \phi$ degrees above the horizon, A coincides with M. When $a > \phi$ we see from Figure 9 that

$$MA = R \tan \frac{1}{2}(-(\phi - a)) = -R \tan \frac{1}{2}(\phi - a),$$

since it holds that $\tan(-x) = -\tan x$. From this observation we see that also in case $a > \phi$, formula (2.2) is still applicable.

2.4 Projection of the azimuthal circles

For the projection of the azimuthal circles we first focus on the so-called *first vertical*, which runs through the West and East point, and through the zenith. From Figure 10 it is seen that it holds that

$$MA = R \tan \frac{1}{2}(90^\circ + \phi)$$

and that

$$MB = R \tan \frac{1}{2}(90^\circ - \phi).$$

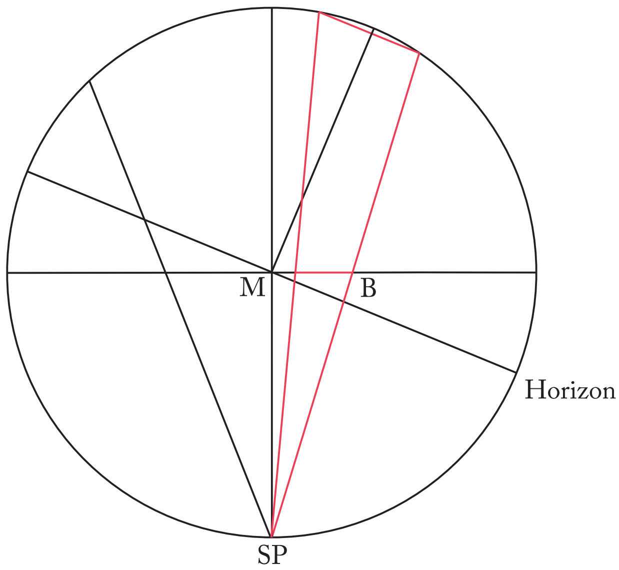

Figure 9: Projection of the altitude circle $a > \phi$ degrees above the horizon for an observer at latitude ϕ.

For the y-component of the center C_V we have, using the same identities (2.1) as before, that

$$
\begin{aligned}
&-\frac{R}{2}\left[\tan\frac{1}{2}(90^\circ+\phi)-\tan\frac{1}{2}(90^\circ-\phi)\right]\\
&=-\frac{R}{2}\left[\tan\frac{1}{2}(90^\circ-\frac{1}{2}(90^\circ-\phi))-\tan\frac{1}{2}(90^\circ-\phi)\right]\\
&=-\frac{R}{\tan(90^\circ-\phi)}=-R\tan\phi.
\end{aligned}
$$

Then we have that $C_V = (0, -R\tan\phi)$.

For the projections of the other azimuthal circles we use that stereographic projection is a conformal mapping, meaning that angles are preserved under the mapping. From this property it follows that the angles between two azimuthal circles on the celestial sphere are the same as the angles between its respective projections in the plane. An angle between two circles is defined as the angle between the two tangent lines at the point where the circles intersect. Further we assume it as a fact that the centers of the projections of the azimuthal circles are all on one line perpendicular to the meridian and through C_V. This fact can be proven by using the properties of stereographic projection.

In Figure 11 the azimuthal circle α degrees North of East is drawn. From this figure we derive that for the distance between the projection Z of the zenith and C_V it holds that

$$ZC_V = R\tan\phi + R\tan\frac{1}{2}(90^\circ-\phi).$$

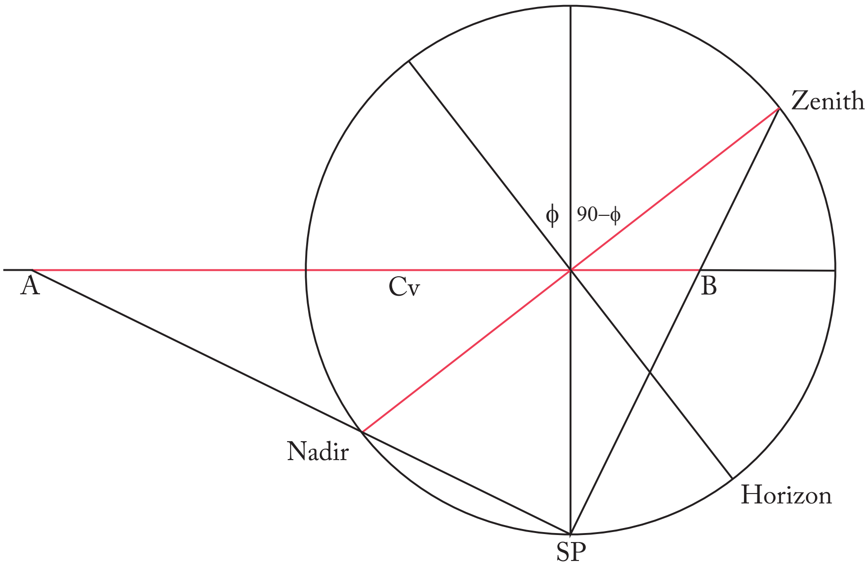

Figure 10: Projection of the first vertical.

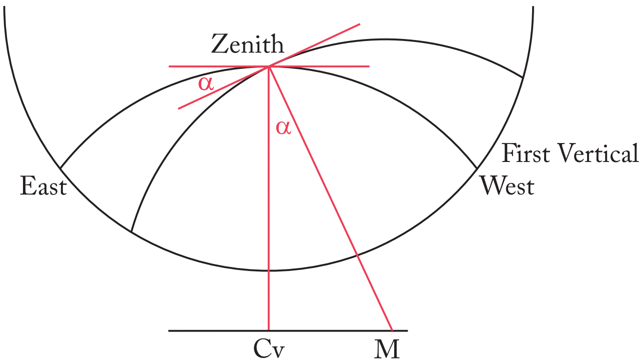

Figure 11: The azimuthal circle α degrees North of East.

Note here that the location of the projection of the zenith, that is the projection of the 'altitude circle' 90 degrees above the horizon, is easily derived by using formula (2.2):

$$Z = (0, R\tan\frac{1}{2}(90^\circ - \phi)).$$

Then the center M of the projection of the azimuthal circle α degrees North of East is located at

$$M = ((R\tan\phi + R\tan\frac{1}{2}(90^\circ - \phi))\tan\alpha, -R\tan\phi).$$

For the radius R_α of the projection we have that

$$R_\alpha = \frac{R\tan\phi + R\tan\frac{1}{2}(90^\circ - \phi)}{\cos\alpha}.$$

In Figure 12 the reconstructed astrolabe plate is drawn for the latitude of Esfahan. The plate is combined with the outer rim, and the inscripitions are in Arabic.

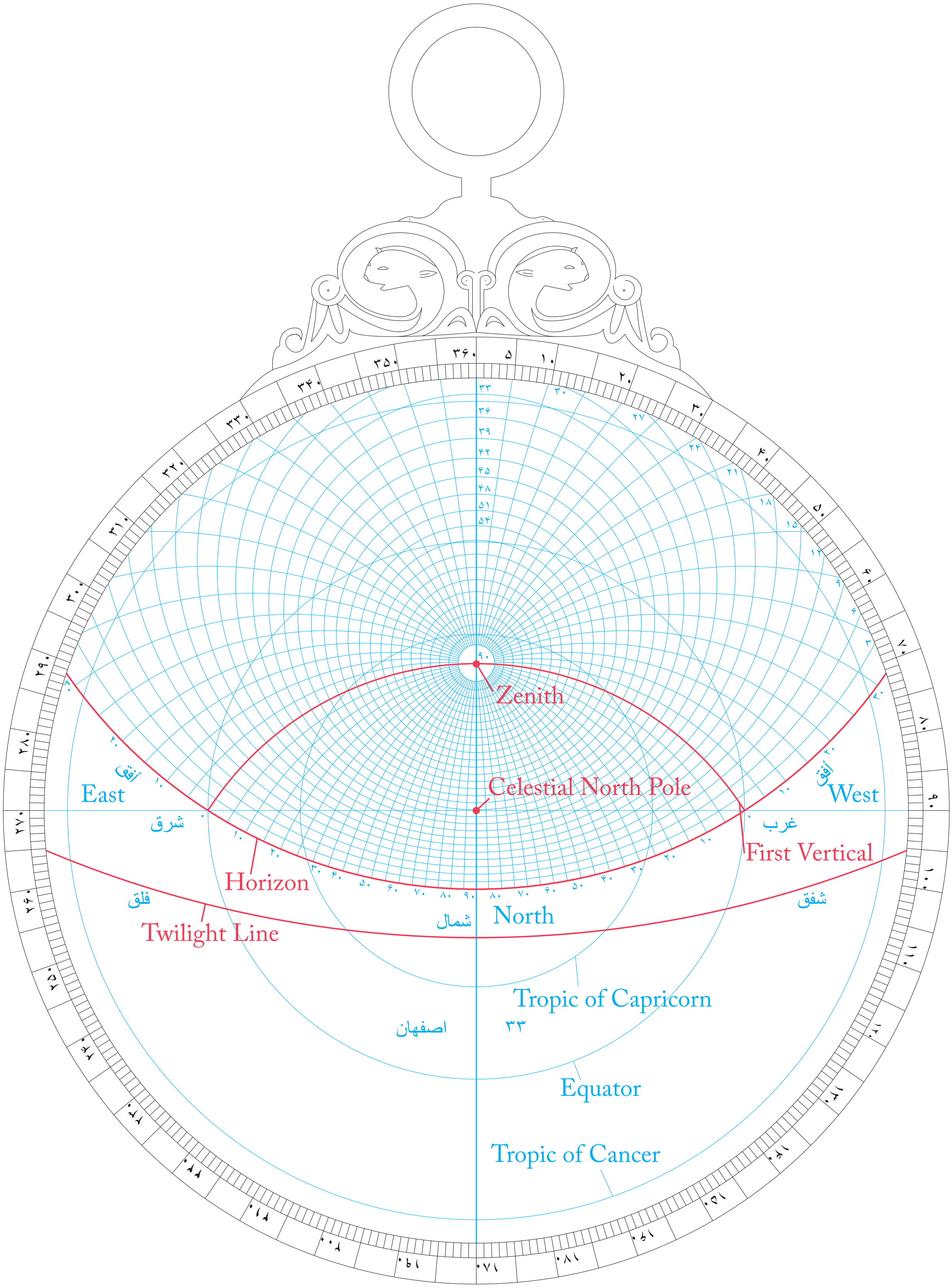

Figure 12: A plate suitable for the latitude for Esfahan combined with the outer rim.

2.5 The Spider

The spider contains the stereographic projections of the *ecliptic*, which is the apparent one year trajectory of the sun along the sky, and of 33 astrolabe stars. The astrolabe stars are the same as on the astrolabe of Al-Khujandī. The positions of the stars are recomputed for the year 2000, showing the effect of precession of the equinoxes if the model is compared to the original astrolabe of Al-Khujandī, see Table 2. The precession is about 15 degrees in a 1000 year interval. In the model, the position of a star is indicated by a dot in the middle of a small circle.

The projection of the ecliptic is easily calculated from Figure 13. From it we see that

$$MA = R\tan\frac{1}{2}(90^\circ - \epsilon)$$

and that

$$MB = R\tan\frac{1}{2}(90^\circ + \epsilon).$$

The projection of the ecliptic is in fact nothing more that the projection of the horizon for an observer at the polar circle: $\phi = 90^\circ - \epsilon$. For the position of the center C of the projection we find $C = (0, R\tan\epsilon)$, since it holds for the y-component of the center C, using again identities (2.1), that

$$\begin{aligned}
&\frac{R}{2}\left[\tan\frac{1}{2}(90^\circ + \epsilon) - \tan\frac{1}{2}(90^\circ - \epsilon)\right] \\
&= \frac{R}{2}\left[\tan\frac{1}{2}(90^\circ - \frac{1}{2}(90^\circ - \epsilon)) - \tan\frac{1}{2}(90^\circ - \epsilon)\right] \\
&= \frac{R}{\tan(90^\circ - \epsilon)} = R\tan\epsilon.
\end{aligned}$$

The position of the sun can be estimated using the fact that the sun moves through the twelve zodiacal signs, into which the ecliptic is divided, in the course of one year. Every sign is divided into 30 degrees. The sun moves with a velocity of approximately one degree per day.

For the projections of the stars on the plane of the celestial equator, we first recall that all stars are projected from the center of the Earth onto the celestial sphere by a straight line. In Figure 13 the projection of a declination circle with declination δ is drawn. From it we see that it is projected onto a circle with center $(0, 0)$ and radius $R\tan\frac{1}{2}(90^\circ - \delta)$. This circle is in the plane characterized by the (x, y)-coordinates:

$$x = -R\cos\alpha\tan\frac{1}{2}(90^\circ - \delta), \qquad y = -R\sin\alpha\tan\frac{1}{2}(90^\circ - \delta),$$

where the angle α is the right ascension of the star. By convention, we have that $\alpha = 0$ is the point in the sky where the sun crosses the celestial equator at the March equinox. This point marks the beginning of spring and is known as the first point of the zodiacal sign Aries. In the table the star positions (x, y) are listed for $R = 1$.

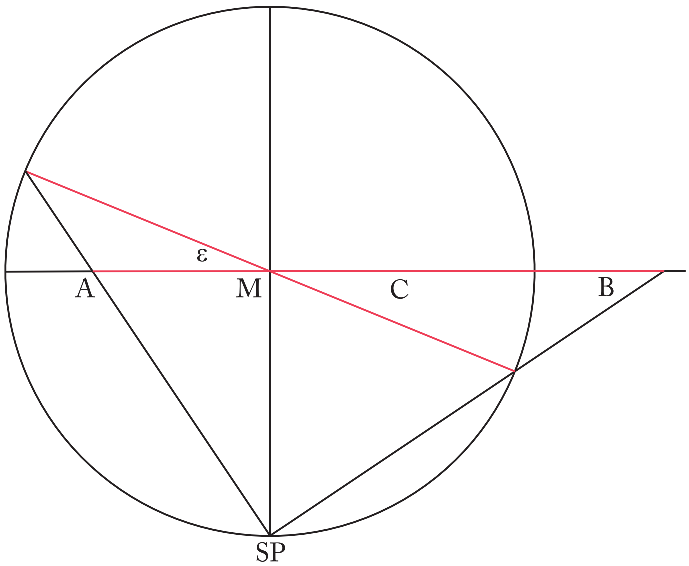

Figure 13: Projection of the ecliptic.

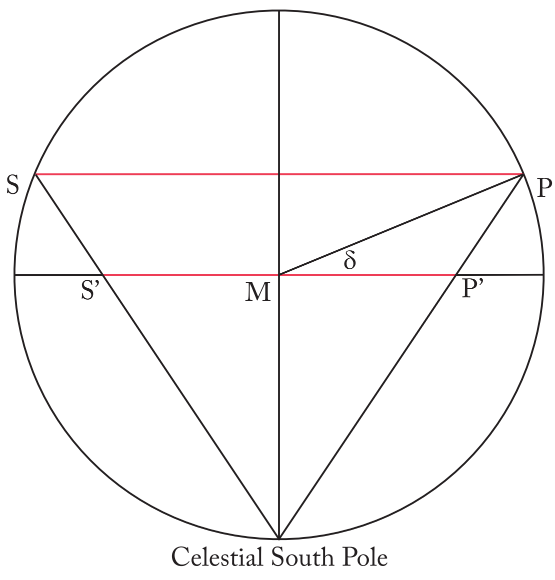

Figure 14: Projection of the declination circle with declination δ.

Name	Star	δ	t	x	y
Sirrah	α Andromedae	$29°5'$	0^h8^m	-0.5877	-0.0205
Caph	β Cassiopeiae	$59°9'$	0^h9^m	-0.2757	-0.0108
Deneb Kaitos	ι Ceti	$-12°49'$	1^h25^m	-1.1662	-0.4581
Baten Kaitos	ζ Ceti	$-10°20'$	1^h51^m	-1.0609	-0.5581
Algol	β Persei	$40°57'$	3^h8^m	-0.3112	-0.3336
Aldebaran	α Tauri	$16°31'$	4^h36^m	-0.2675	-0.6969
Rigel	β Orionis	$-8°12'$	5^h14^m	-0.2302	-1.1313
Capella	α Aurigae	$46°00'$	5^h17^m	-0.0754	0.3969
Alnilam	ϵ Orionis	$-1°12'$	5^h36^m	-0.1067	-1.016
Betelgeuze	α Orionis	$7°24'$	5^h55^m	-0.0192	-0.8783
Sirius	α Canis Maioris	$-16°43'$	6^h45^m	0.2623	-1.3186
Procyon	α Canis Minoris	$5°13'$	7^h39^m	0.3822	-0.8290
	ρ Puppis	$-24°18'$	8^h8^m	0.8207	-1.3134
Alphard	α Hydrae	$-8°40'$	9^h28^m	0.9172	-0.7166
Regulus	α Leonis	$11°58'$	10^h8^m	0.7154	-0.3803
Tania Australis	μ Ursae Maioris	$41°29'$	10^h22^m	0.4101	-0.1869
Gienah	γ Corvi	$-17°33'$	12^h16^m	1.3617	0.0952
Spica	α Virginis	$-11°10'$	13^h25^m	1.1340	0.4410
Alqaid	η Ursae Maioris	$49°19'$	13^h48^m	0.3303	0.1683
Arcturus	α Bootis	$19°11'$	14^h16^m	0.5893	0.3975
	ζ Bootis	$13°44'$	14^h41^m	0.5992	0.5072
Gemma	α Corona Borealis	$26°43'$	15^h35^m	0.3644	0.4969
	β Serpentis	$15°25'$	15^h46^m	0.4203	0.6351
Antares	α Scorpii	$-26°26'$	16^h29^m	0.6241	1.4883
Ruticulus	β Herculis	$21°29'$	16^h30^m	0.2606	0.6292
Rasalhague	α Ophiuci	$12°34'$	17^h35^m	0.0873	0.7967
Vega	α Lyrae	$38°47'$	18^h37^m	-0.0770	0.4731
Altair	α Aquilae	$8°52'$	19^h51^m	-0.3652	0.7742
	ϵ Delphini	$11°18'$	20^h33^m	-0.5076	0.6439
Deneb	α Cygni	$45°17'$	20^h41^m	-0.2658	0.3130
Gienah	ζ Cygni	$30°14'$	21^h13^m	-0.4287	0.3826
Deneb Algedi	δ Capricorni	$-16°7'$	21^h47^m	-1.1121	0.7292
Scheat	β Pegasi	$28°5'$	23^h04^m	0.5821	0.1451

Table 2: The star positions for the year 2000.

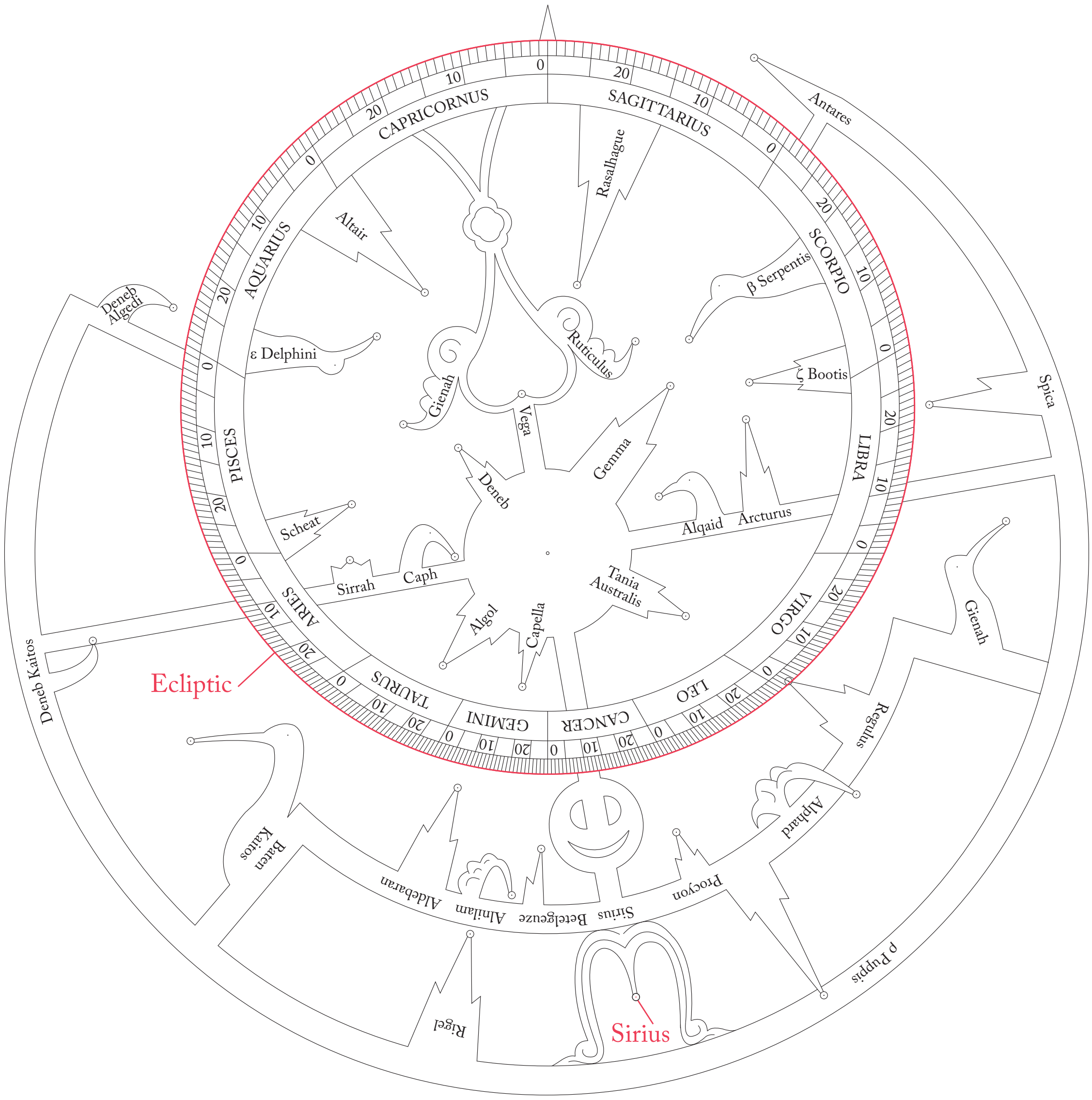

Figure 15: The ecliptic and the star Sirius highlighted on the spider.

3 How to Use the Astrolabe

This section is about the many uses of the astrolabe.

3.1 Use of the Astrolabe

Let us start with the back side of the astrolabe, which containes a metal strip with two sights and a pointer, the so-called *alidade*. The alidade can be used to measure the altitude of the sun or a star in degrees, when the astrolabe is suspended vertically. The altitude can then be read off on a circular scale.

For any day, the position of the sun in the ecliptic can be marked on the spider. The position of the sun can be estimated using the fact that the sun moves through the twelve zodiacal signs, into which the ecliptic is divided, in the course of one year. Every sign is divided into 30 degrees. The sun moves with a velocity of approximately one degree per day. In Table 3 the zodiacal signs and their corresponding months are listed. The spider can now be set to represent the actual position of the celestial constellations with respect to the horizon. The astrolabe can now be used to calculate the local time and the length of daylight. The astrolabe can also be used as a compass.

For the determination of the local time and also for the calculation of the length of a certain day, note that the pointer of the spider indicates a number on the rim. A full rotation of the spider corresponds to 24 hours, so 1 degree of rotation corresponds to 4 minutes of time. By rotating the spider, one can determine the interval of time between the moment of observation and, for example, sunset, noon, and sunrise. The direction of the sun can be read off, by means of the azimuthal circles, for example 10 degrees South of East.

Also the direction of *qibla*, i.e. the direction of prayer to Mecca, could be calculated by means of the astrolabe, when knowing the longitude of the observer. However, the astrolabe was not used for this practice.

3.1.1 Calculating the length of daylight

In this example we calculate the length of daylight on 17 October by using the astrolabe. You can try to do the same calculation for another day, for instance your anniversary day.

1. First we need to know the position of the sun in the ecliptic on 17 October. From Table 3 we see that the sun is in degree 26 of the zodiacal sign Libra on 17 October. (If it might happen that your anniversary day is in degree 31 of a particular sign, write 30).

2. Now we can mark the position of the sun on the ecliptic on the spider. Be sure to mark it on the outer rim of the ecliptic.

3. Now we rotate the spider in such a way that the sun is positioned at the Eastern horizon, to simulate sunrise. We can read off the position of the pointer: 213.

Aries	الحمل	March 21	- April 19
Taurus	الثور	April 20	- May 20
Gemini	الجوزأ	May 20	- June 20
Cancer	السرطان	June 21	- July 22
Leo	الاسد	July 23	- August 22
Virgo	السنبلة	August 23	- September 22
Libra	الميزان	September 23	- October 22
Scorpio	العقرب	October 23	- November 21
Sagittarius	القوس	November 22	- December 21
Capricornus	الجدي	December 22	- January 19
Aquarius	الدلو	January 20	- February 18
Pisces	الحوت	February 19	- March 20

Table 3: The signs of the zodiac in Latin and in Arabic with the corresponding dates.

4. Next we rotate the spider such that the sun is at the Western horizon, where it sets. The position of the pointer is then 19.

5. The difference between the position of the pointer at sunset and the position of the pointer at sunrise is 166 degrees. If you encounter a negative difference for your own anniversary day, add 360 degrees to the position of the pointer at sunset.

6. Next step is to recalculate this difference in degrees to a real time difference. Recall that one full rotation of the spider corresponds to 24 hours. So 15 degrees corresponds to 1 hour, and 1 degree to 4 minutes. This in a length of daylight on 17 October of 11 hours and 4 minutes.

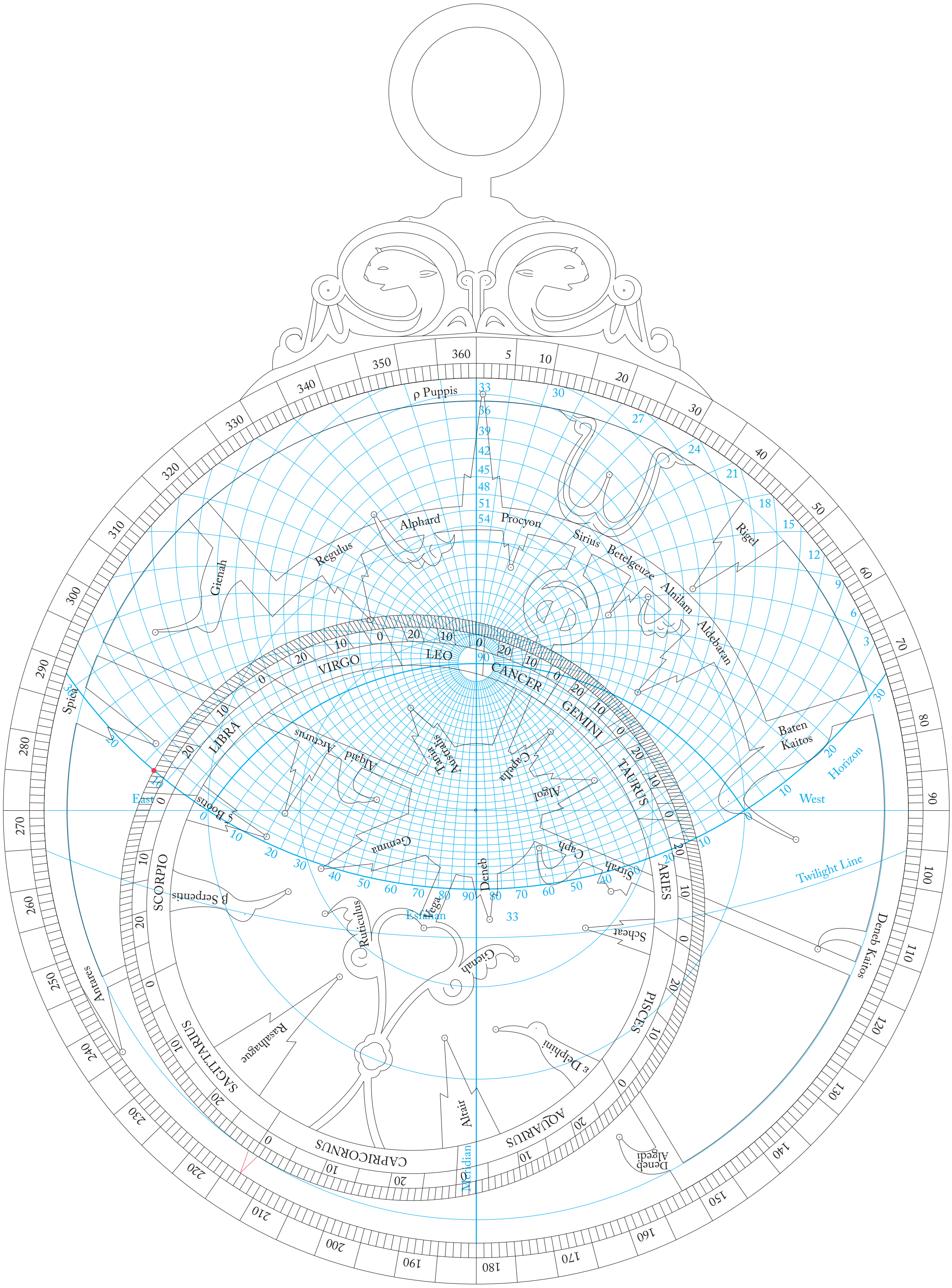

Figure 16: The sun positioned at the Eastern horizon.

3.1.2 Calculating the local time and direction

In this example we use the astrolabe to calculate the true local solar time, and use it to determine the direction of the sun.

Suppose that we have measured with the alidade on the back side of the astrolabe that the sun is 9 degrees above the horizon. We have done the measurement in the afternoon of 17 October. You can again try to do the same calculation for your anniversary day.

1. Because the measurement is done in the afternoon, the sun is close to the Western horizon, where the sun sets. The position of the pointer at that moment is 8 degrees.

2. Now we use the fact that the sun directs at noon (12.00 true local solar time) exactly to the South. So we rotate the spider in such a way that the sun is positioned at the meridian. The position of the pointer at noon is 296 degrees.

3. The difference between the position of the pointer at the moment that the sun is 9 degrees above the horizon and the position of the pointer at noon is 72 degrees.

4. This difference we recalculate to 4 hours and 48 minutes. So the true local solar time at the moment that the sun is 9 degrees above the horizon is 16 hours and 48 minutes.

5. The direction of the sun at the moment that the sun is 9 degrees above the horizon we read off using the azimuthal lines as being 19 degrees South of West.

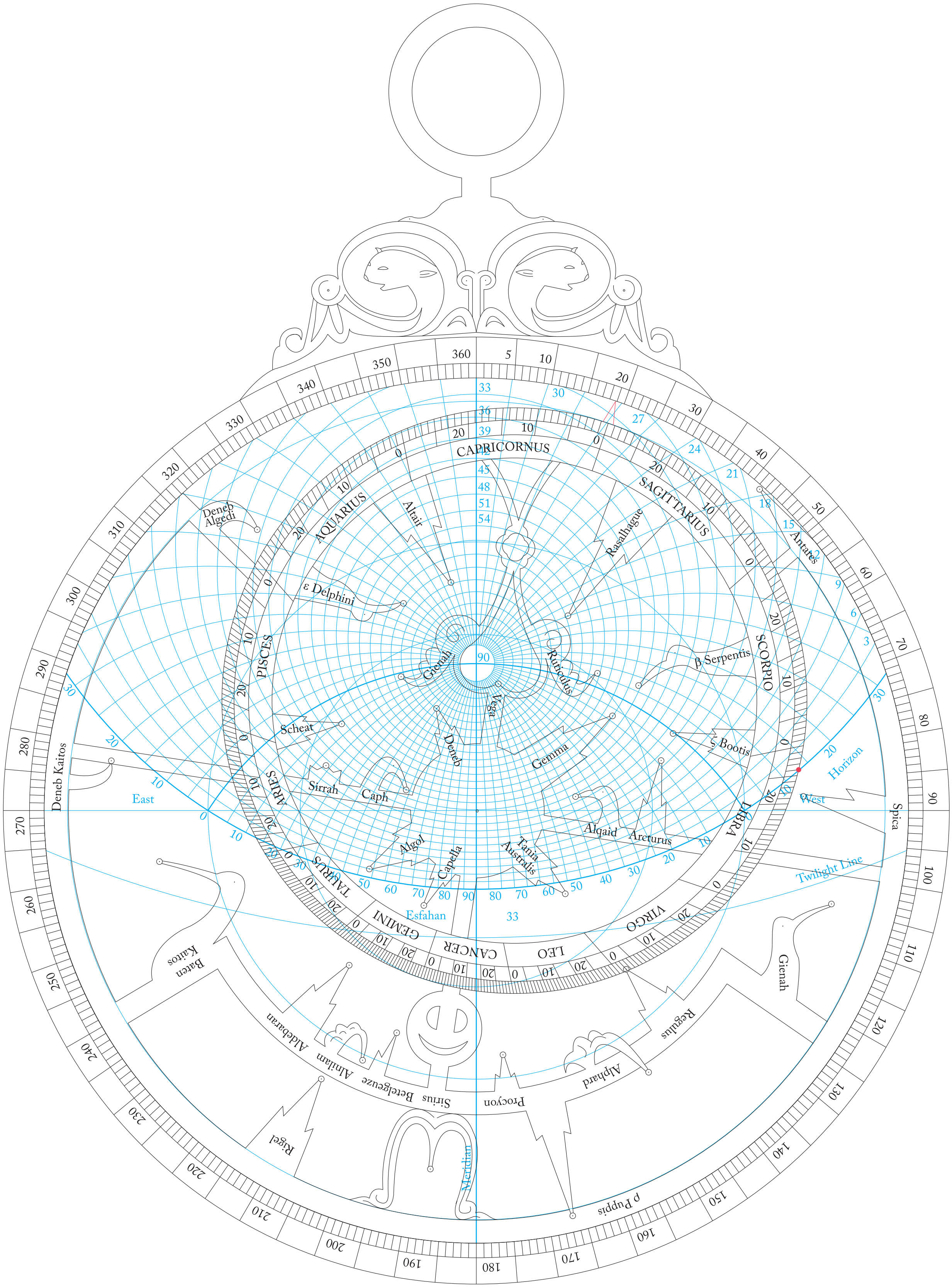

Figure 17: The sun positioned at the Western horizon.

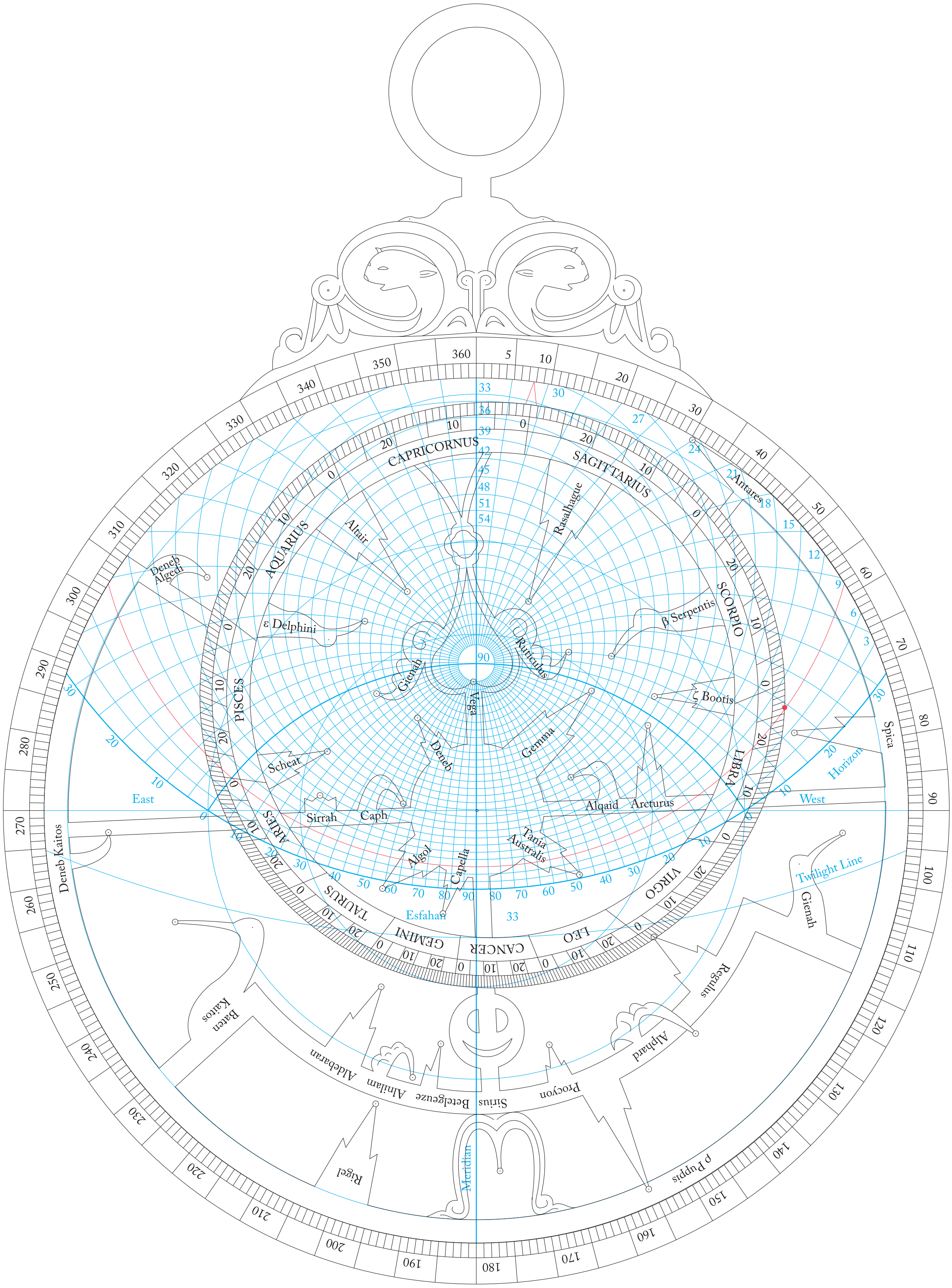

Figure 18: The sun positioned 9 degrees above the horizon during the afternoon.

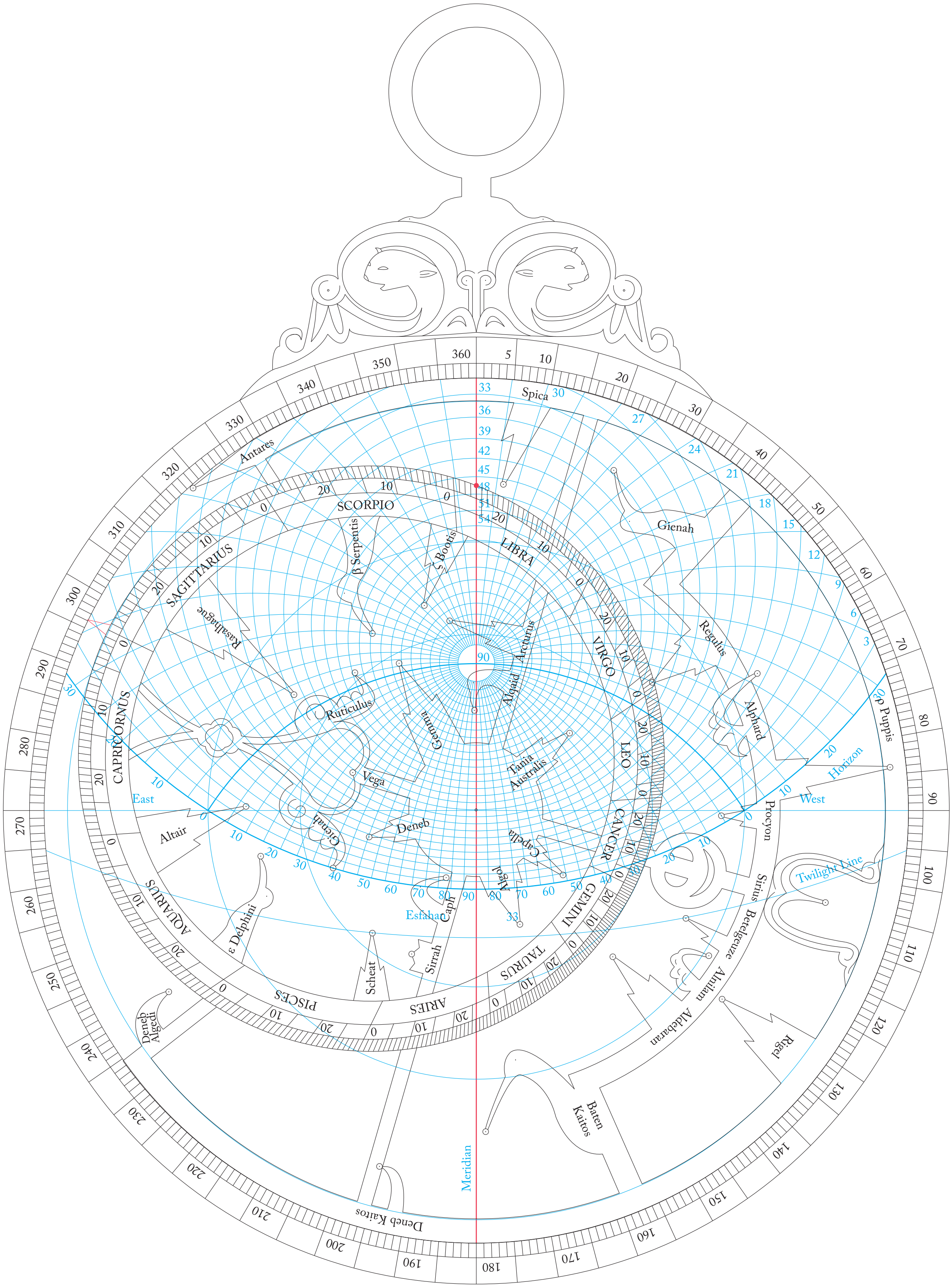

Figure 19: The sun positioned at the meridian.

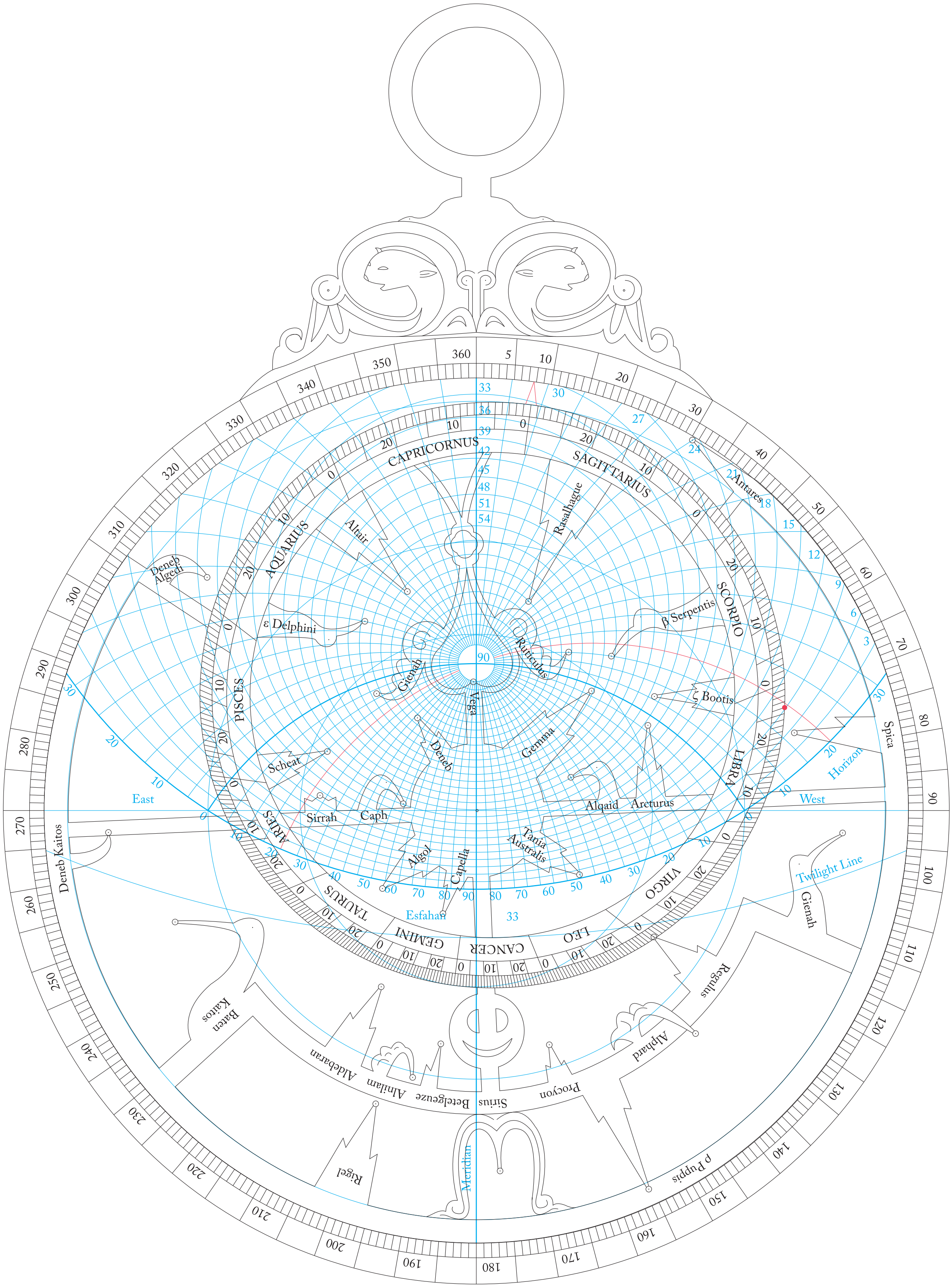

Figure 20: Using the azimuthal lines to read off the direction of the sun.

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